Productivity and Misallocation Dynamics under Dominant Currency Paradigm*

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April 9, 2025

Preliminary

Abstract

US monetary policy has a significant impact on the global economy, with recent literature emphasizing the dollar's role as the dominant currency. This paper examines how US monetary policy affects productivity in non-US economies through allocative efficiency. We develop a two-country model featuring dollar dominance in trade and misallocations from markup heterogeneity. When US monetary policy tightens, dollar appreciation affects non-US economies through two channels. First, it lowers the marginal costs in dollars of non-US exporters, causing large export firms with high markups that underproduce relative to the efficient allocation to incompletely pass through these changes, thereby increasing their markups further. Second, in domestic markets, dollar appreciation raises import prices, easing competitive pressures for local producers. Both effects reallocate resources from large, high-markup firms to small, low-markup firms, worsening allocative efficiency. Using plant-level data from Chile and Colombia, where trade is predominantly invoiced in dollars, we provide evidence of factor reallocation from high-markup to low-markup firms following US monetary tightening.

JEL Codes: D24, D21, D61, E12, E25, F43, F44, F12

*I am grateful to Ariel Burstein, David Baqaee, and Oleg Itskhoki for their comments. I would also like to thank the participants of the macroeconomics seminar at UCLA.

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1 Introduction

International trade is predominantly invoiced in a handful of "dominant currencies," with the US dollar playing a central role in this system. Gopinath et al. (2020) formalized this concept as the dominant currency paradigm (DCP). Under DCP, firms set their export prices in dollars regardless of their trading partners and tend to adjust these dollar prices infrequently. Previous literature has extensively discussed how dollar fluctuations under DCP affect world trade volume and terms of trade [1].

This paper argues that under DCP, dollar appreciation or depreciation endogenously changes global productivity through shifts in allocative efficiency. This critical effect has remained unexplored in existing literature and does not manifest in conventional DCP models or international business cycle frameworks. The concept of allocative efficiency is intimately connected to firm-level markups and pass-through rates—the extent to which firms transmit cost changes to prices—which we explore in detail throughout this paper.

It is well established that pass-through rates decrease as firm size increases. Theoretically, markups rise rapidly with firm size (Atkeson and Burstein (2008)). When facing marginal cost increases due to exchange rate movements, larger firms absorb these shocks by adjusting their markups, while smaller firms directly pass these shocks through to prices. In other words, higher markups correlate with lower pass-through rates. This relationship, sometimes referred to as Marshall's third law, finds strong empirical support in Amiti et al. (2020).

An economy characterized by heterogeneous markups is inherently inefficient because large firms with higher markups underproduce relative to the efficient allocation in the steady state. However, under pass-through heterogeneity, large firms may reduce their markups in response to lower marginal costs, potentially moving the economy closer to efficient allocation. This mechanism has recently been studied in sticky price models within closed economies (e.g.,Baqaee et al. (2021); Reinelt and Meier (2020); Fujiwara and Matsuyama (2023)).

We extend these arguments to open economies under DCP. A crucial distinction between open and closed economies is that firms compete with foreign counterparts that face different marginal costs in both domestic and foreign markets. Moreover, because prices are sticky in dollars under DCP, exchange rates have asymmetric effects on dollar-denominated marginal costs for US and non-US firms. Our analysis reveals that heterogeneous markups, heterogeneous pass-through rates, and price stickiness under DCP all play essential roles in determining allocative efficiency.

[1] For other DCP papers, see Gopinath and Itskhoki (2021); Itskhoki (2021); Mukhin (2022).

Our framework allows us to decompose the effects on allocative efficiency into two distinct channels. The pass-through channel measures how heterogeneous markup changes affect allocative efficiency. The expenditure switching channel captures how resources are reallocated across firms as relative prices change, and can be further subdivided into the DCP channel (related to price stickiness in dollars) and the marginal cost channel (reflecting changes in underlying factor prices). Allocative efficiency improves if either channel leads to increased relative size of previously underproducing firms.

Our model consists of two countries—the US and a representative non-US economy—where firms compete monopolistically under DCP within a non-parametric generalized Kimball (1995) demand system introduced by Matsuyama and Ushchev (2017). In the steady state, heterogeneity in firm distribution and markups creates misallocation. The model's only asymmetry is the invoice currency of trade; all other aspects remain symmetric. We approximate the model around this inefficient steady state, which is crucial because, as in Gopinath et al. (2020), approximating around the efficient equilibrium would entirely eliminate the first-order effects of changes in misallocation[2].

We investigate how dollar appreciation associated with tightening US monetary policy affects productivity. Dollar appreciation reduces the dollar-denominated marginal costs of non-US exporters while simultaneously raising the prices of imported goods in non-US economies. Consequently, for non-US firms, competition eases in both domestic and foreign markets, leading to a reallocation of production factors away from underproducing firms and worsening allocative efficiency. In contrast, since US firms reference their marginal costs in their home currency (the dollar), exchange rate fluctuations have limited impact on their markups. Instead, as argued by Baqaee et al. (2021); Reinelt and Meier (2020); Fujiwara and Matsuyama (2023), marginal costs (domestic factors) in the US decline as demand falls with tightening US monetary policy, reducing aggregate productivity. Thus, through different mechanisms, US monetary policy can influence aggregate productivity in both US and non-US economies.

Finally, using plant-level data for Chile and Colombia, where the invoice currency for trade is almost exclusively the dollar, we provide evidence of factor reallocation from high markup firms to low markup firms in response to U.S. monetary tightening.

[2]Our endogenous misallocation dynamic is related to Marshall's second and third laws of demand. The second law states that the price elasticity of demand increases with its price, meaning that more productive firms in our model will have higher markup rates. The third law is a term coined by Matsuyama and Ushchev (2022), which states that, in addition to the second law, the rate of increase in price elasticity falls with its price, implying that firms with higher markup have lower pass-through rates in our setting. We will discuss this point in detail in Section 2.

Related Literature

Much of the allocative efficiency literature has focused on the long-run impact on productivity (Hsieh and Klenow (2009); Bartelsman et al. (2013); Baqaee and Farhi (2020) for a closed economy and Arkolakis et al. (2018); Edmond et al. (2015) for open economy). Recently, however, the importance of allocative efficiency in the business cycle has been pointed out (e.g., Baqaee et al. (2021); Reinelt and Meier (2020)). Our study is related to misallocation over the business cycle, but our model is a two-country model and thus considers firms with different marginal costs competing in the same market. We derive the first dynamic equation for the misallocation under a class of flexible utility functions. It can be decomposed into terms related to each of Marshall's third law and heterogenous markup, each of which has a clear interpretation.

This paper is also related to the international productivity co-movement literature. The idea that markup affects a change in productivity goes back to Hall (1988); Basu and Fernald (2002). In an economy that uses imported intermediate goods, if there is a markup, the imported intermediate input raises value added more than its input value. However, in GDP calculations, only nominal imports (=costs) are subtracted. Thus, even if there is no change in domestic technology, an increase in the input of imported intermediate goods mechanically increases GDP and productivity, measured as the Solow residual[3].

However, these are different from the mechanism presented in this paper. The mechanism is a statistical bias in the productivity measurement that also arises under a uniform markup in the presence of imported intermediate goods. The mechanism we present is not a statistical bias but a real change in aggregate productivity due to the reallocation of factors across firms.

Finally, Huo et al. (2020) argue that adjusting for capacity utilization can explain some of the productivity correlation, but they point out that when productivity correlation is weakened by adjusting for capacity utilization, a new correlation mechanism is needed to drive the co-movement of the international business cycle. They also use the assumption that aggregate productivity is entirely exogenous to identify capacity utilization, but the "capacity utilization" they identify could be due to the endogenous reallocations we present, in which case the assumption for identification would not hold.

In Section 2, we derive the model. We explain how the heterogeneity of markup and pass-throughs under the generalized Kimball demand leads to the misallocation

[3] Burstein and Cravino (2015), Baqaee and Farhi (2019) derive an analytical expression of how the import effect TFP with imported intermediate and markup. Empirically, Gaillard and de Soyres (2021) recently found that this mechanism can explain some of the productivity correlations across countries.

in steady-state. We then explain how the log-linearized model around the inefficient equilibrium affects TFP at the first-order. We then describe how asymmetries in currency price stickiness shape misallocation dynamics. In Section 3, we run simulations using a calibrated model consistent with Amiti et al. (2020)'s pass-thorough estimates. We confirm that US monetary policy shocks cause changes in non-US productivity but that non-US monetary policy shocks do not affect US productivity. Section 4 extends the model to HSA utility. Finally, Section 5 presents evidence using micro and macro data.

2 Modeling Framework

Our model consists of two countries, US and non-US, where firms compete monopolistically under DCP with a non-parametric generalized Kimball (1995) demand system introduced by Matsuyama and Ushchev (2017). In a steady state, there is heterogeneity in firm distribution and markup, and thus misallocation. Since the two countries are perfectly symmetric except for the currency regime, we describe only the home equation. The asymmetric part of the model is the assumption of a pricing regime, DCP, where the prices of both countries' exports and imports are determined in dollars. Note that when foreign goods are consumed, they must be converted to the home currency.

Finally, to streamline the exposition, we introduce the following notation: For two variables $x_\theta > 0$ and $z_\theta > 0$, we define the x-weighted expectation

$$E_x\left[z_\theta\right] = \frac{\int_0^1 z_\theta x_\theta d\theta}{\int_0^1 x_\theta d\theta}$$

We denote the sales share density of firm type θ by

$$\lambda_\theta = \frac{p_\theta y_\theta}{\int_0^1 p_\theta y_\theta d\theta}$$

2.1 Model Setup

Below, a capital letter denotes the level of the variable and a lower letter denotes the log deviation from the steady state. Variables with $*$ denote foreign variables. As for nominal variables, they are basically expressed in the currency of the country in which they are. In other words, variables marked with $*$ are in US dollars, and all others are home currency (non-dollar). However, when a home currency is converted into dollars, a \$ sign may be added to the variable. The nominal exchange rate $\mathcal{E}_t$ is the price of dollars in terms of home currency; hence an increase in $\mathcal{E}_t$ is a nominal devaluation of the home currency.

Households

A representative home household maximizes the discounted expected utility over consumption and labor:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\eta} C_t^{1-\eta} - \frac{1}{1+1/\nu} L_t^{1+\frac{1}{\nu}} \right)$$

where $1/\eta$ represents the intertemporal elasticity of substitution, and ν is the Frisch elasticity of labor supply.

The budget constraint is given by:

$$P_t^{CPI} C_t + \frac{B_{t+1}}{R_t} = W_t L_t + B_t + \Pi_t$$

where P_t^{CPI} is the consumer price index, W_t is the nominal wage, B_t is the quantity of local government bonds that will pay out one unit of home currency in the next period, and R_t is the nominal gross interest rate.

Demand for Varieties and Price Indices

The home households allocate their within-period consumption expenditures $P_t^{CPI} C_t$ between home and foreign goods. Consumption bundles C_{ht} and C_{ft} consist of different varieties of goods indexed by $\theta \in [0, 1]$.

$$P_t^{CPI} C_t = P_{ht} C_{ht} + P_{ft} C_{ft} = \int_0^1 \left(P_{h\theta t} C_{h\theta t} + P_{f\theta t} C_{f\theta t} \right) d\theta$$

Notice that we consider domestic and international products with the same θ as differentiated variety. The utility from each consumption bundle is implicitly defined using a generalized Kimball (1995) aggregator, $\Upsilon\left(\right)$ as follows.

$$\int_0^1 \left[(1-\gamma) \Upsilon_\theta \left(\frac{C_{h\theta,t}}{(1-\gamma) C_t} \right) + \gamma \Upsilon_\theta \left(\frac{C_{f\theta,t}}{\gamma C_t} \right) \right] d\theta = 1 \tag{1}$$

where $\gamma \in [0, 1/2)$ represents the trade openness. $C_{h\theta,t}$ and $C_{f\theta,t}$ are the consumption of variety θ from home and foreign, and Υ_θ is an increasing and concave function. Following Itskhoki and Mukhin (2021); Gopinath et al. (2020), it is adjusted so that domestic and foreign firms with the same θ face identical positions in the demand curve. This makes the misallocation equation tractable since firms with the same θ impose identical markups for the domestic market and export in the steady states. However, we allow the domestic and export markups to move differently *around the steady states*.

The optimal expenditure allocation solution yields the following inverse demand schedules for domestic and foreign goods for variety θ:

$$\frac{P_{h\theta t}}{P_t} = \Upsilon'_\theta \left(\frac{C_{h\theta,t}}{(1-\gamma)\, C_t} \right) \tag{2}$$

$$\frac{P_{f\theta t}}{P_t} = \Upsilon'_\theta \left(\frac{C_{f\theta,t}}{\gamma C_t} \right) \tag{3}$$

where the price aggregator P is given by: (4)

$$P = \frac{P_t^{CPI}}{\int \left[\Upsilon'_\theta \left(\frac{C_{h\theta t}}{(1-\gamma) C_t} \right) \frac{C_{h\theta t}}{C_t} + \Upsilon'_\theta \left(\frac{C_{f\theta t}}{\gamma C_t} \right) \frac{C_{f\theta t}}{C_t} \right] d\theta} \tag{4}$$

where P_t^{CPI} is an ideal price index for consumer.

The above expression indicates that the price index $P_t \neq P_t^{CPI}$ in general. Importantly, if we assume a heterogeneous firm, the price aggregator P_t does not coincide with the consumer price index P_t^{CPI}, even at the first-order. We will argue that the economic interpretation when we derive the first-order approximation of this expression later.

Finally, household utility maximization yields the Euler equation and labor supply conditions as follows:

$$i_t = \eta \left(\mathbb{E}_t c_{t+1} - c_t \right) + \pi_{t+1}^{CPI}$$

$$\eta c_t + \zeta l_t = w_t - p_t^{CPI}$$

where p_t^{CPI} and π_t^{CPI} are the consumer price index and the inflation rate, respectively. The detailed definitions are described later.

Firms.

We assume that the economy has a continuum of firms indexed by θ. Firm-specific productivity A_θ is exogenously given and does not change. For convenience, sort in order of decreasing productivity. Since firms have linear production technology using labor and produce goods for home and export with the same technology, the production function is given by:

$$\begin{cases} Y_{\theta ht} = A_\theta L_{\theta ht} & \text{for home} \\ Y^*_{\theta ht} = A_\theta L^*_{\theta ht} & \text{for export} \end{cases}$$

Note that firms with the same θ in different countries have the same productivity but do not produce homogeneous goods. Unlike Itskhoki and Mukhin (2021); Gopinath et al. (2020), we assume that there are no intermediate goods and that labor is the only factor of production. The impact of intermediate goods is discussed in Section 4.

Desired markup and pass-through

We introduce three statistics related to the shape of the demand curve: demand elasticity, markup, and pass-through. First, demand elasticities for firm type θ facing are given by

$$\frac{Y'_\theta\left(\frac{C_{h\theta}}{(1-\gamma)C}\right)}{-Y''_\theta\left(\frac{C_{h\theta}}{(1-\gamma)C}\right)\frac{C_{h\theta}}{(1-\gamma)C}} = \frac{Y'_\theta\left(\frac{C_{f\theta}}{\gamma C}\right)}{-Y''_\theta\left(\frac{C_{f\theta}}{\gamma C}\right)\frac{C_{f\theta}}{\gamma C}} = \sigma_\theta$$

where $c_{f\theta}$, $c_{f\theta}$ and C are steady-state value of each variables.

We introduce the desired markup

$$\mathcal{M}_\theta = \frac{\sigma_\theta}{\sigma_\theta - 1}$$

This is the markup under flexible prices. This value is the same for all firms in the New Keynesian model with the usual CES assumptions. However, under the Kimball demand function system, the markup value faced changes depending on the residual demand position.

Finally, the firm's desired pass-through price ρ_θ is the elasticity of the firm's optimal price with respect to its marginal cost when the economy-wide aggregate quantity is held constant (partial equilibrium). The desired pass-through of firm θ can be expressed as follows.

$$\begin{aligned}\rho_\theta = \frac{\partial log P^{flex}_{f\theta}}{\partial log mc} &= \frac{1}{1 + \frac{\frac{C_{f\theta}}{\gamma C^*}\mathcal{M}'_\theta}{\mathcal{M}_\theta}\sigma_\theta} \\ = \frac{\partial log P^{flex}_{h\theta}}{\partial log mc} &= \frac{1}{1 + \frac{\frac{C_{h\theta}}{(1-\gamma)C}\mathcal{M}'_\theta}{\mathcal{M}_\theta}\sigma_\theta}\end{aligned}$$

The above equality arises from the symmetry of the model in steady state. As we explain later, the dynamics of productivity depend critically on the distribution of firms, markup, and pass-throughs. We formally define Marshall's second and third laws.

Definition 1. Marshall's second and third laws.

Marshall's second law of demand assumes:

$$\mathcal{M}'_\theta > 0$$

$$\rho_\theta < 1$$

Marshall's third law assumes, in addition to the second law, the following conditions:

$$\rho'_\theta < 0$$

The second law implies that the larger the firm, the higher the markup and the incomplete pass-through. Note that the second law allows for a constant pass-through less than one. Marshall's third law, on the other hand, is a stronger version of the second law and further assumes that pass-through decreases as markup increases. Marshall's third law of demand has strong empirical support (see, for example, empirical estimates of pass-through by firm size by Amiti et al. (2019)) and holds in a variety of models (Atkeson and Burstein (2008)). We assume that Marshall's third law holds in our paper.

Firm level Pricing under DCP

Firms with type θ maximize profits by providing identical goods to their home and foreign markets. Under DCP, US firms set prices in dollars for both domestic and exports and in dollars for exports. Assume that firms set prices a la Calvo with a probability of changing price next period equal $(1-\delta)$.

The optimal reset price for US firms satisfies the following first-order conditions:

$$p^{\tilde{*}}_{f\theta t} = (1-\beta\delta)\left(\rho_\theta mc^*_{t+j} + (1-\rho_\theta)\, p^*_{t+j}\right) + \beta\delta\mathbb{E}_t p^{*\tilde{}}_{f\theta t+1}$$

$$\underbrace{p^{\tilde{\$}}_{f\theta t}}_{\text{Reset Price in Dollar}} = (1-\beta\delta)\left(\rho_\theta \underbrace{mc^*_{t+j}}_{\text{MC in Dollar}} + (1-\rho_\theta)\, p^{\$}_{t+j}\right) + \beta\delta p^{\$\tilde{}}_{f\theta t+1}$$

Prices for the domestic market are determined by a weighted average of own marginal cost and price index with desired pass-through, ρ_θ. By Marshall's third law of demand,

ρ_θ declines with firm size, so larger firms are more price sensitive to their competitors. In exports, the US also sets prices in dollars, so the competitive price index referenced is in dollars. When the US tightens monetary policy, even if the exchange rate changes significantly, the dominant change in the reset price will be attributed to changes in marginal cost. When downward pressure is exerted on mc^*_{t+j} due to reduced demand, the markup relatively increases for larger firms with smaller pass-throughs.

The optimal reset prices (log deviation from the steady state) for domestic, $p_{\tilde{h\theta t}}$ and export prices, $p^{\tilde{*}}_{h\theta t}$ are as follows:

$$p_{\tilde{h\theta t}} = (1-\beta\delta)\left(\rho_\theta mc_t + (1-\rho_\theta)\underbrace{p_t}_{\text{Home competetor's price}}\right) + \beta\delta\mathbb{E}_t p_{h\tilde{\theta t}+1}$$

$$p^{\tilde{*}}_{h\theta t} = (1-\beta\delta)\left(\rho_\theta \underbrace{[mc_t - e_t]}_{\text{MC in Dollar}} + (1-\rho_\theta)\underbrace{p^*_t}_{\text{US competetor's price}}\right) + \beta\delta\mathbb{E}_t p^{*\tilde{}}_{h\theta t+1}$$

Next, consider the pricing of non-US firms. Although the reset price for domestic exports appears to be apparently similar, the rigid US export price in dollars increases the price of imports in the home currency as the dollar appreciates in the dollar. This change pushes up p_t. The easing of the competitive environment causes large firms to increase their markup relative to their domestic counterparts.

On the other hand, the exporter's optimal price is the discounted weighted average of marginal cost in dollars and the price index. Consider the case where the dollar appreciates (e increases) and the marginal cost in dollars decreases. As the firm size increases, the proportion referring to the decreased marginal cost in dollars decreases, so the reset price of the markup realized if the price could be reset is relatively larger. Thus, as firm size increases, the markup increase will be relatively larger.

Combining this equation with the case where prices cannot be changed, the firm-level price dynamics for US and non-US type θ firms are as follows:[4].

$$E\left[\left(p_{h\theta,t}-p_{h\theta,t-1}\right)\right]=\beta E\left[\left(p_{h\theta,t+1}-p_{h\theta,t}\right)\right]+\psi\left(-p_{h\theta,t}+\rho_{\theta}mc_{t}+\left(1-\rho_{\theta}\right)p_{t}\right) \tag{5}$$

$$E\left[p^{*}_{h\theta,t}-p^{*}_{h\theta,t-1}\right]=\beta E\left[p^{*}_{h\theta,t+1}-p^{*}_{h\theta,t}\right]+\psi\left(-p^{*}_{h\theta,t}+\rho_{\theta}\left(mc_{t}-e_{t}\right)+\left(1-\rho_{\theta}\right)p^{*}_{t}\right) \tag{6}$$

$$E\left[p^{*}_{f\theta,t}-p^{*}_{f\theta,t-1}\right]=\beta E\left[p^{*}_{f\theta,t+1}-p^{*}_{f\theta,t}\right]+\psi\left(-p^{*}_{f\theta,t}+\rho_{\theta}mc^{*}_{t}+\left(1-\rho_{\theta}\right)p^{*}_{t}\right) \tag{7}$$

$$\begin{aligned}E\left[p_{f\theta,t}-p_{f\theta,t-1}\right]-\Delta e_{t}=&\beta E\left[p_{f\theta,t+1}-p_{f\theta,t}-\Delta e_{t-1}\right]+\\&\psi\left(-\left(p_{f\theta,t}-e_{t}\right)+\rho_{\theta}mc^{*}_{t}+\left(1-\rho_{\theta}\right)\left(p_{t}-e_{t}\right)\right)\end{aligned} \tag{8}$$

where $\psi=\frac{1-\delta}{\delta}\left(1-\beta\delta\right)$.

By weighting these firm-level price dynamics, various aggregate-level dynamics can be derived. The following lemma is the result of a log-linear approximation of two price indices around a steady state.

Lemma 1. *Around an asymmetric equilibrium of firms, the expenditure share on foreign goods equals γ, and the log-linear approximation to the two domestic price indies, p_t^{CPI} and p_t are a weighted average of varieties prices:*

$$p_t=\left(1-\gamma\right)p^{\sigma}_{ht}+\gamma p^{\sigma}_{ft}$$

$$p^{CPI}_t=\left(1-\gamma\right)p_{ht}+\gamma p_{ft}$$

where $p_{xt}=E_{\lambda}\left[p_{x\theta t}\right]$ and $p^{\sigma}_{xt}=E_{\lambda\sigma}\left[p_{x\theta t}\right]$ with $x\in\{h,f\}$.

This lemma explains the difference in the weights used to aggregate the two price indexes: p_t^{CPI} is aggregated by the sales share of firm type θ, whereas p_t uses the sales share multiplied by the elasticity of substitution faced by the firm as a weight. Intuitively, p_t summarizes the information on competitors' prices that firms should consider when pricing. Since it is easier to take market share from firms facing higher elasticities, it is optimal to give more weight than the firm's sales share.

Using Lemma 2, the dynamic equation for p_t can be obtained by weighting and averaging the price dynamics equations for individual firms. For example, p_t can be derived as the weighted average of the firm-level price dynamics equations, which is

[4] The weighted average of the price unchanged and the reset price. For example, the following are the expected prices for home firms for their home country. We have

$$\mathbb{E}_t p_{h\theta t}=\delta p_{h\theta t-1}+\left(1-\delta\right)p_{\tilde{h\theta t}}$$

$(6) \times (1-\gamma) + (9) \times \gamma$ and then rearranged, taking the expectation by $E_{\lambda\sigma}$. NKPC for home market price index is

$$\pi_t - \gamma \Delta e_t = \psi \left(E_{\lambda\sigma} \left[\rho_\theta \right] \left[(1-\gamma) \, mc_t + \gamma \left(w_t^* + e_t \right) - p_t \right] \right) + \beta \left[\mathbb{E}_t \pi_{t+1} - \gamma \Delta e_{t+1} \right]$$

2.2 Allocative Efficiency

We then proceed to examine the dynamics of allocative efficiency under the DCP regime, which is the central concept of this paper. First, we introduce the concept of allocative efficiency and discuss its relationship to firm-level micro-moments. Next, we discuss the relationship between macro price information and allocative efficiency (macro sufficient statistics)

Define *an aggregate productivity*A_t as an aggregate output per unit of labor, so that

$$A_t = \frac{Y_t}{L_t}$$

where $Y_t = \int_0^1 \left(y_{h\theta t} + y_{h\theta t}^* \right) d\theta$

From this representation, the change in Y_t can be decomposed into changes in L_t and A_t.

$$y_t = a_t + l_t$$

It is important to notice that we assumeA_θ of individual firms is exogenously fixed. That is, the aggregated A_t moves not because of changes in the technology of particular firms but because of the reallocation of factors of production (labor).

To establish the relationship between changes in markup and aggregate productivity, we apply the main results of Baqaee and Farhi (2019). The above formula is a more general one that includes Hsieh and Klenow (2009) formula for allocative efficiency as a special case; Hsieh and Klenow (2009)'s formula assumes that demand is CES and firm productivity and markup are jointly lognormally distributed, but neither assumptions do not apply to our model[5]. Under the assumptions of a symmetric country and the absence of intermediate goods, we rearrange the formulas to obtain the following proposition that governs the relationship between micro-level statistics of firms and allocative efficiency.

Lemma 2. *Micro-level Sufficient Statistics for Allocative Efficiency:*

[5] The statistics in Hsieh and Klenow, 2009 depend on changes in markup dispersion and elasticity of substitution, σ: $\Delta \log A = -(\sigma/2) \, Var \left(\log \mathcal{M}_\theta \right)$. This equation holds only if demand is CES and firm productivity and markup dispersion are jointly lognormally distributed, which is generally not the same as the covariance of Lemma 1.

A change in aggregate productivity can be summarized by the covariance between markup level and labor changes.

$$a_t = Cov_\lambda \left[-\left(\underbrace{\bar{\mathcal{M}}/\mathcal{M}_\theta}_{Relative\ Markup} \right), \underbrace{(1-\gamma)\, l_{h\theta t}}_{For\ home} + \underbrace{\gamma l^*_{h\theta t}}_{For\ export} \right] \tag{9}$$

Aggregate productivity increase iff labor are reallocated to high-markup firms for whatever reasons.

Despite the open economy setting, the formula for the covariance between labor change and markup is similar to Baqaee et al. (2021) under a closed economy. This is because the reallocation of factors between tasks within the same firm has no first-order effect on productivity since the level of markup in the steady state is the same for domestic and exports. In other words, only the reallocation of labor between firms matters for allocative efficiency. This implies that productivity increases when labor is reallocated to firms with a higher (relative to average) markup.

Lemma 2 implies that the covariance between the reallocation of factors of production and the (relative) initial markup is a sufficient statistic as a result of the change in allocative efficiency. The above formula is a more general one that includes Hsieh and Klenow (2009) formula for allocative efficiency as a special case; Hsieh and Klenow (2009)'s formula assumes that demand is CES and firm productivity and markup are jointly lognormally distributed, but neither assumptions do not apply to our model. [6]

Productivity Dynamics

Combining Lemma 2 with firm-level price dynamics allows us to express productivity as a function of macro-level aggregate prices. The next proposition is an equation that links the macro-level price index to the exchange rate and productivity responses. To this end, we introduce a useful lemma.

Lemma 3. *Marshall's laws and covariance between firm-level statistics:*

$$Cov_\lambda \left(\frac{\bar{\mathcal{M}}}{\mathcal{M}_\theta}, \sigma_\theta \right) \geq 0$$

where equality holds when the elasticities are all equal (=CES).

[6] The statistics in Hsieh and Klenow, 2009 depend on changes in markup dispersion and elasticity of substitution, σ: $\Delta \log A = -(\sigma/2)\, Var(\log \mathcal{M}_\theta)$. This equation holds only if demand is CES and firm productivity and markup dispersion are jointly lognormally distributed, which is generally not the same as the covariance of Lemma 1.

If Marshall's third law holds,

$$Cov_{\lambda}\left(\frac{\sigma_{\theta},}{E_{\lambda}\left[\sigma_{\theta}\right]},\rho_{\theta}\right) > 0$$

The first covariance implies that the inverse of the markup (relative to the mean) is positively correlated with the price elasticity. Since the markup is $\mathcal{M}_{\theta} = \frac{\sigma}{\sigma_{\theta}-1}$, this inequality always holds, and the equality holds when the elasticities are all equal (=CES). The second covariance implies that the covariance between price elasticity (relative to its average) and pass-through is positive. In this case, the incomplete pass-through alone is insufficient, requiring that the pass-through decreases with firm size.

Proposition 1. *The allocative efficiency follows the second-order difference equation:*

$$\begin{aligned} a_t = {} & \frac{\kappa}{\psi}a_{t-1} + \frac{\kappa}{\psi}\beta a_{t+1} \\ & + \kappa \left\{ \begin{array}{c} \chi\left[mc_t - \left\{(1-\gamma)\,p_t + \gamma\left(p_t^* + e_t\right)\right\}\right] + \\ \xi\left[mc_t - \left(mc_t^* + e_t\right)\right] - \omega\left[\Delta e_t - \beta\Delta e_{t+1}\right] \end{array} \right\} \end{aligned}$$

$$\begin{aligned} a_t^* = {} & \frac{\kappa}{\psi}a_{t-1}^* + \frac{\kappa\beta}{\psi}a_{t+1}^* \\ & + \kappa \left\{ \begin{array}{c} \chi\left[mc_t^* - \left\{(1-\gamma)\,p_t^* + \gamma\left(p_t - e_t\right)\right\}\right] + \\ \xi\left[mc_t^* - \left(mc_t - e_t\right)\right] + \omega\left[\Delta e_t - \beta\Delta e_{t+1}\right] \end{array} \right\} \end{aligned}$$

where $\kappa = \frac{\psi}{1+\beta+\psi}$, $\psi = \frac{1-\delta}{\delta}\left(1-\beta\delta\right)$ *and* $\bar{\mathcal{M}} = E_{\lambda}\left[\mathcal{M}_{\subseteq}{}^{-1}\right]^{-1}$ *is a aggregate markup.*

$$\xi = 2E_{\lambda\sigma}\left[\rho_{\theta}\right]\gamma\left(1-\gamma\right)Cov_{\lambda}\left(\frac{\bar{\mathcal{M}}}{\mathcal{M}_{\theta}},\sigma_{\theta}\right) \geq 0$$

$$\omega = \frac{1}{\psi}\left(1-\gamma\right)\gamma Cov_{\lambda}\left(\frac{\bar{\mathcal{M}}}{\mathcal{M}_{\theta}},\sigma_{\theta}\right) \geq 0$$

Under the third law of demand,

$$\chi = \bar{\mathcal{M}}Cov_{\lambda}\left(\frac{\sigma_{\theta},}{E_{\lambda}\left[\sigma_{\theta}\right]}\rho_{\theta}\right) > 0$$

These expressions have clear economic interpretations. Let us focus on the non-US productivity a_t. We call the first term as *pass-through channel.*

$$\bar{\mathcal{M}} Cov_{\lambda} \left(\frac{\sigma_\theta}{E_\lambda[\sigma_\theta]}, \rho_\theta \right) \left[mc_t - \left\{ (1-\gamma)\, p_t + \gamma \left(p_t^* + e_t \right) \right\} \right]$$

The increase in marginal cost/reduction in the price index to be exposed measures the increase in competition. The covariance between demand elasticity and pass-through is positive, indicating that allocative efficiency increases as firms with larger markups lower their markups in response to more intense competition.

The second and third terms measure the effect of the expenditure-switching channel on productivity in response to changes in the competitive environment. To understand these terms, we assume $\beta = 0$ and set the lag term is zero. Then we have an intuitive static expression as follows:

$$\gamma \left(1-\gamma\right) Cov_{\lambda} \left(\frac{\bar{\mathcal{M}}}{\mathcal{M}_\theta}, \sigma_\theta \right) \left\{ 2E_{\lambda\sigma} \left[\rho_\theta \right] (1-\delta) \left[mc_t - \left(mc_t^* + e_t \right) \right] - \delta e_t \right\}$$

These terms measures the effect of expenditure switching when the markup is given. The covariance between markup and price elasticity is positive, meaning that as competition increases, firm facing lower elasticities will increase their relative size. Since each firm's markup is inversely proportional to the elasticity of demand, high markup firms expand relative to low markup firms, increasing allocative efficiency. The marginal cost differential affects allocative efficiency even in the absence of price rigidity, but the exchange rate is present only when there is price rigidity in DCP[7].

To clarify the role of Marshall's third law, it is useful to look at changes in world aggregate productivity.

Corollary 1. *World Allocative Efficiency:*

$$\tilde{a}_t = \frac{\kappa}{\psi} \tilde{a}_{t-1} + \frac{\kappa}{\psi} \beta \tilde{a}_{t+1} + \kappa \bar{\mathcal{M}} Cov_{\lambda} \left(\frac{\sigma_\theta,}{E_\lambda \left[\sigma_\theta \right]}, \rho_\theta \right) \left\{ \; mc_t + mc_t^* - \left[p_t + p_t^* \right] \; \right\}$$

where $\tilde{a}_t = a_t + a_t^*$.

The change in the productivity of the two countries added together is due solely to the reallocation effect associated with the difference in pass-through. In other words, this

[7] These second and third terms are related to *the Darwinian effect* of Baqaee et al. (2020). They studied the reallocation effect associated with market expansion, where all firms face identical marginal costs. Increased competition is measured by a decrease in the price index due to the love of variety associated with firm entry. On the other hand, our model has firms with different marginal costs competing in the same market, and the difference in marginal costs between the two countries measures the intensity of competition.

effect may increase productivity in both countries. Conversely, the part related to expenditure switching (the difference in marginal costs between the two countries) will always favor one country. In other words, the pass-through channel will play an important role in moving allocative efficiency in the same direction in the two countries.

We next argue that endogenous productivity responses are muted under assumptions commonly used in the existing literature.

Example 1. CES demand system:

The CES function is a special case of the generalized Kimball demand function: Since the CES function has common elasticity and perfect pass-through, the two covariances, $Cov_\lambda\left(\frac{\bar{\mathcal{M}}}{\mathcal{M}_\theta}, \sigma_\theta\right)$ and $Cov_\lambda\left(\frac{\sigma_\theta,}{E_\lambda[\sigma_\theta]}, \rho_\theta\right)$ are both *globally* zero. Therefore,

$$a_t = a_t^* = 0$$

Next, we will see a non-CES case.

Example 2. An approximation around a symmetric steady state under non-CES demand:

For example, Gopinath et al. (2020) made a first-order approximation around a symmetric equilibrium under the Kimball demand system. In the steady state, the demand elasticities and markup for all firms are identical since all firms are at the same demand curve position. Thus, the two covariances $Cov_\lambda\left(\frac{\bar{\mathcal{M}}}{\mathcal{M}_\theta}, \sigma_\theta\right)$ and $Cov_\lambda\left(\frac{\sigma_\theta,}{E_\lambda[\sigma_\theta]}, \rho_\theta\right)$ are *locally* zero. Moreover, if the position of the demand curve for all firms is the same in initial equilibrium, it will move exactly the same in response to shocks. Thus, this term remains zero at second and higher orders. Therefore,

$$a_t = a_t^* = 0$$

2.3 Closing the Model

Finally, we describe the conditions necessary to close the required model. First, we obtain the standard UIP condition from the perfect risk-sharing. We also impose the usual market clearing condition. Monetary policy follows the Taylor Rule. The other conditions for solving the model follow the standard open economy NK model. We will provide the list of equations after linear approximation in the appendix.

3 Model Simulation

In this section, we calibrate the model and perform quantitative simulations to argue that misallocated channels cause productivity co-movements. The structure of this section is as follows. First, we calibrate the Kimball demand function non-parametrically, following Baqaee et al. (2021). Next, we present the impulse response of productivity to US and non-US monetary policy shocks.

3.1 Firm Level Statistics

The sufficient statistics of the firm distribution needed for the model are $Cov_\lambda\left(\frac{\bar{\mathcal{M}}}{\mathcal{M}_\theta}, \sigma_\theta\right)$ and $Cov_\lambda\left(\frac{\sigma_\theta,}{E_\lambda[\sigma_\theta]}, \rho_\theta\right)$, $E_{\lambda\sigma}[\rho_\theta]$ and the average markup $\bar{\mathcal{M}} = E_\lambda\left[\mathcal{M}_{\subseteq}{}^{-1}\right]^{-1}$. One way to compute these statistics is to impose a parametric function (e.g., Klenow and Willis (2016)) on Kimball demand, an approach popular in the literature. The parametric approach works well when one is interested in the pass-through of the average firm, as in Gopinath et al. (2020); Itskhoki and Mukhin (2021). However, as Baqaee et al. (2020) emphasize, the function of Klenow and Willis (2016) could not capture the heterogeneity of firm pass-through well. As discussed in the previous section, heterogeneity in pass-through under the third law is important for endogenous productivity. We, therefore, adopt the non-parametric calibration method of Baqaee et al. (2020) derived from Amiti et al. (2019)'s pass-through estimates and firm size distributions. See the appendix for specific procedures for calibrations.

Other parameters are set as follows. For the Frisch elasticity, we set $\zeta = 0.2$ to match the micro-level estimates of Chetty et al. (2011) , and the other conventional parameters are those of Galí (2015), with Calvo parameters, Taylor rule coefficient, and discount factor. The monetary policy shock follows AR (1) and the inertia parameter, ρ_i is set to 0.7. Finally, the value of the home bias $(1-\gamma)$ was set to 0.7 according to Gopinath et al. (2020) . The following table summarizes the values of the parameters.

$\bar{\mathcal{M}}$	$\frac{1}{\eta}$	ξ	δ	ϕ_y	ϕ_π	β	ρ_i	$(1-\gamma)$
1.15	1	0.2	0.5	0.5/4	1.5	0.99	0.7	0.7

Table 1: Calibrated Parameters

3.2 Impulse response

Using the calibrated model described above, we give positive monetary policy shocks of 0.25 basis points for the US and non-US under the DCP assumption. We report annualized values. Figure 1 shows the US and non-US impulse responses to US monetary policy. First, US monetary tightening leads to a stronger dollar, which lowers the dollar-denominated marginal cost of non-US exporters and thus raises the markup of large non-US firms. On the other hand, in response to US monetary tightening, large US firms raise their markup because of lower marginal costs. This shifts the relative factor allocation from large to small firms in both countries. This fact can be confirmed by checking the impulse response of markup dispersion from changes in firm-level markup. In other words, US monetary policy shocks worsen US and non-US aggregate productivity. For the other variables, the response is similar to the previous model. As US monetary policy tightens, US GDP, consumption, and prices decline. Non-US will also decline but to a smaller extent. CPI Inflation will rise as import prices rise.

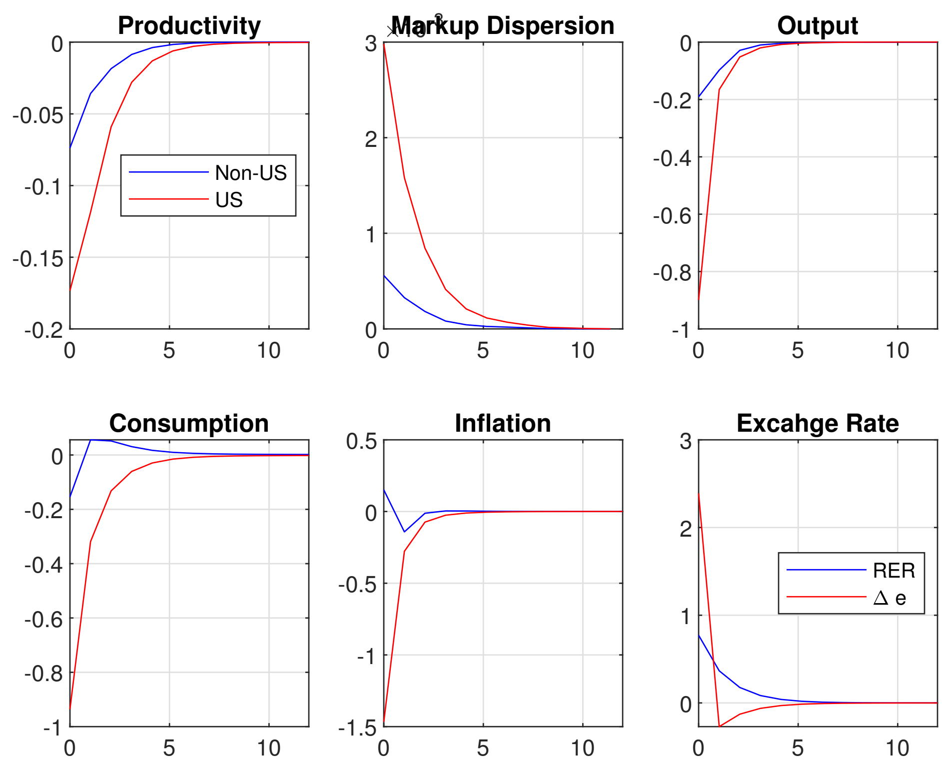

Figure 1: The response of the US and non-US economies to a 0.25 basis point US monetary policy shock. All responses are expressed in annualized percentiles; RER stands for Real Exchange rate.

Next, Figure 2 is the response of non-US to monetary policy shocks, but highlights the role of DCP setup. Non-US monetary policy shocks cause reallocation from large to small firms through a decrease in home country marginal costs, as is the case for US monetary policy shocks. However, there is a striking difference in the US productivity response. While the exchange rate fluctuates in response to monetary policy shocks, large US firms do not change their prices significantly under price stickiness because they trade in their home currency, the dollar. In other words, markup do not change significantly either. Thus, in contrast to the large impact of US monetary policy shocks on non-US productivity, non-US monetary policy shocks have little impact on US productivity. Recall that the invoice currency assumption is the only source of heterogeneity between the two countries in this model. Even though country sizes are exactly the same in both countries for simplicity, differences in the invoice currency assumption shape differences in the response of productivity to monetary policy.

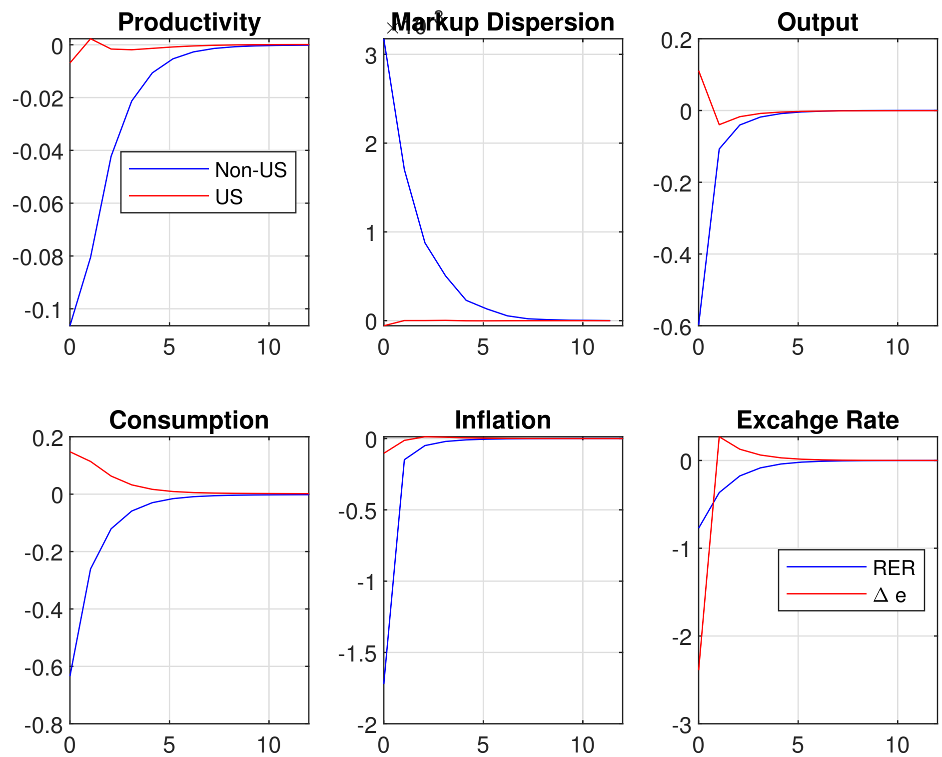

Figure 2: Responses of the US and non-US economies to a 0.25 basis point non-US monetary policy shock. All responses are expressed in annualized percentiles; RER stands for Real Exchange rate.

4 Extension

4.1 Extension with HSA Demand System

We built our model under a non-parametric generalized Kimball demand (HDIA). Our results can be easily applied to another flexible demand system proposed by Matsuyama and Ushchev (2017), the Homothetic Single Aggregator (HSA). Since most results, including the dynamic equation in Proposition 1, are characterized using the shape of the non-parametric demand function, the exact same equation holds under HSA. The only exception is the price index P, which we will briefly discuss below. Let us replace HDIA demand system (1) with the following HSA demand system. The expenditure shares of each variety θ are[8]

$$\frac{p_{h\theta t}y_{h\theta t}}{P_t^{CPI}C_t} = (1-\gamma)\, s_\theta\left(\frac{p_{h\theta}}{P}\right)$$

$$\frac{p_{f\theta}y_{f\theta}}{P_t^{CPI}C_t} = \gamma s_\theta\left(\frac{p_{f\theta}}{P}\right)$$

where $c_{x\theta t}$ and $p_{x\theta t}$ ($x \in \{h, f\}$) are consumption and price of the variety θ. s_θ is decreasing function. The price aggregator P is implicitly defined by the condition that the sum of expenditure shares equals 1.

$$\int_0^1 \left[(1-\gamma)\, s_\theta\left(\frac{p_{h\theta}}{P}\right) + \gamma s_\theta\left(\frac{p_{f\theta}}{P}\right)\right] d\theta = 1 \tag{10}$$

Applying the derivative of implicit function to (10), for all $\theta \in (0, 1)$ we have

$$\frac{\partial \log P}{\partial \log P_{h\theta}} = \frac{(1-\gamma)\,\frac{p_{h\theta}}{P}s'_\theta\left(\frac{p_{h\theta}}{P}\right)}{\int_0^1\left[(1-\gamma)\,\frac{p_{h\theta}}{P}s'_\theta\left(\frac{p_{h\theta}}{P}\right) + \frac{p_{f\theta}}{P}\gamma s'_\theta\left(\frac{p_{f\theta}}{\gamma P}\right)\right]d\theta}$$

$$\frac{\partial \log P}{\partial \log P_{f\theta}} = \frac{(1-\gamma)\,\frac{p_{f\theta}}{P}s'_\theta\left(\frac{p_{f\theta}}{P}\right)}{\int_0^1\left[(1-\gamma)\,\frac{p_{h\theta}}{P}s'_\theta\left(\frac{p_{h\theta}}{P}\right) + \frac{p_{f\theta}}{P}\gamma s'_\theta\left(\frac{p_{f\theta}}{\gamma P}\right)\right]d\theta}$$

Under these settings, the price index is as follows:

Lemma 4. *Around an asymmetric equilibrium of firms, the expenditure share on foreign goods equals γ, and the log-linear approximation to the two domestic price indies under HSA are:*

$$p_t = \frac{E_\lambda\left[\sigma_\theta\right]\left((1-\gamma)\, p^\sigma_{ht} + \gamma p^\sigma_{ft}\right) - p_t^{CPI}}{\left(E_\lambda\left[\sigma_\theta\right] - 1\right)}$$

$$p_t^{CPI} = (1-\gamma)\, p_{ht} + \gamma p_{ft}$$

where $p_{xt} = E_\lambda\left[p_{x\theta t}\right]$ and $p^\sigma_{xt} = E_{\lambda\sigma}\left[p_{x\theta t}\right]$ with $x \in \{h, f\}$.

Under HDIA, we have $p_t = (1-\gamma)\, p^\sigma_{ht} + \gamma p^\sigma_{ft}$, while in the HSA, it is the weighted

[8] As in HIDA, the home bias parameter gamma is used to adjust the demand curve so that domestic and foreign firms with the same theta are in the same position in the demand curve.

average of the price index in the HDIA and the "CPI". By replacing the price index for the US economy in the same way, our earlier results can be applied directly.

4.2 Discussion on IO Network Extension

In reality, intermediate goods play an important role in the factors of production as well as labor. The introduction of imported intermediate goods gives rise to several conflicting forces. First, an increase in the dollar raises the price of imported intermediate goods, thereby making the home country less competitive and partially offsetting the channel through marginal costs. On the other hand, changes in allocative efficiency are amplified by multiplier effects. On the other hand, in an economy with a markup, a decline in imported intermediate goods has the effect of mechanically lowering measures of productivity. How these effects are offset is ambiguous. See Gaillard and de Soyres (2021); Baqaee and Farhi (2019) for details on the mechanical effects of imported intermediate goods.

5 Empirical Evidence

This section provides empirical evidence for the mechanism described in this paper. We find that reallocation happens within different firm sizes in non-US countries in response to US monetary policy shocks, consistent with the mechanism we propose. On the other hand, evidence on the reallocation induced by US monetary policy within the US is discussed in detail in Baqaee et al. (2021); Reinelt and Meier (2020), which assume a closed economy. For this reason, this paper will focus on non-US empirical analysis.

5.1 Evidence for Micro-Level Reallocation

We first provide evidence of a micro-level response behind the productivity response. Our empirical analysis is motivated by the following equations in Lemma 2 below.

$$a_t = Cov_\lambda \left[-\left(\underbrace{\bar{\mathcal{M}} / \mathcal{M}_\theta}_{Relative\, Markup} \right), \underbrace{(1-\gamma)\, l_{h\theta t}}_{For\, home} + \underbrace{\gamma l^*_{h\theta t}}_{For\, export} \right]$$

This equation indicates that when a reallocation of factors of production from high-markup firms to low-markup firms occurs, aggregate productivity rises. Thus, a testable implication of our model is that production resources will be reallocated from high-markup firms to low-markup firms in response to the tightening of US monetary policy.

Database We use microdata to argue that reallocation occurs from high-markup firms to low-markup firms in response to monetary policy. We use plant-level data for Chile and Colombia. Both countries are ideal for testing our hypotheses most of their export currencies are in dollars, as shown in Table 1 below. The first dataset is the Encuesta Nacional Industrial Anual (ENIA), an annual census of manufacturing in Chile, covering the period 1979-1996. This data covers all Chilean manufacturing industries with ten or more employees, including about 5,000 plants per year. The second data set is the Annual Census of Manufacturing in Colombia, which covers 1981-1991. This data set includes about 7,000 factories per year. Factories with fewer than ten employees were excluded in 1983 and 1984. For markup, we estimate plant-level markup according to Raval (2019). A limitation of these data is that they are annual.

	USD	EUR	Home
Chile export	94.4	3.7	0.3
Colombia export	98.7	0.3	0.7
Chile import	87.5	7.6	2.6

Table 2: Invoicing currency Share of Chile and Colombia from Boz et al. (2022). The dataset includes annual shares of exports and imports invoiced in USD, Euro, and other currencies for 115 countries from 1990 to 2019. We report simple averages of USD, Euro, and home country invoice currency shares for the available data. For Chile, the invoice currencies for exports and imports are available from 2004 to 2019. For Colombia, data are available from 2007 to 2018. Colombian imports are not reported as the percentage of invoice currency of imports is not available.

The specific estimation procedure is as follows. Although our model abstracts from industries, to ensure that reallocations within the same industry affect productivity, we define a High markup firm as one that has a median markup of 3 digits within the industry in each year. We conduct the following local projections using Romer and Romer (2004) monetary policy shock to estimate the relative response of labor in high-markup firms to low-markup firms. We estimate the intersection term of the monetary policy shock and the high-markup firm dummy. The estimation equation includes lag of monetary policy shocks and log worker and plant level fixed cost. We estimated these equations for Chile and Colombia for the available data periods.

$$y_{i,t+h} - y_{i,t-1} = \alpha^h + \alpha_i^h + \gamma \left[\varepsilon_t^{USMP} \times \mathbb{I}\left(i = \text{high markup firm}\right)\right] + \sum_{k=0}^{4} \beta_k^h \varepsilon_{t-k}^{USMP} + \sum_{k=1}^{4} \beta^h dy_{t-k} + u_{it}^h$$

where y_t are $\log\left(L_{i,t}\right)$. We will plot γ.

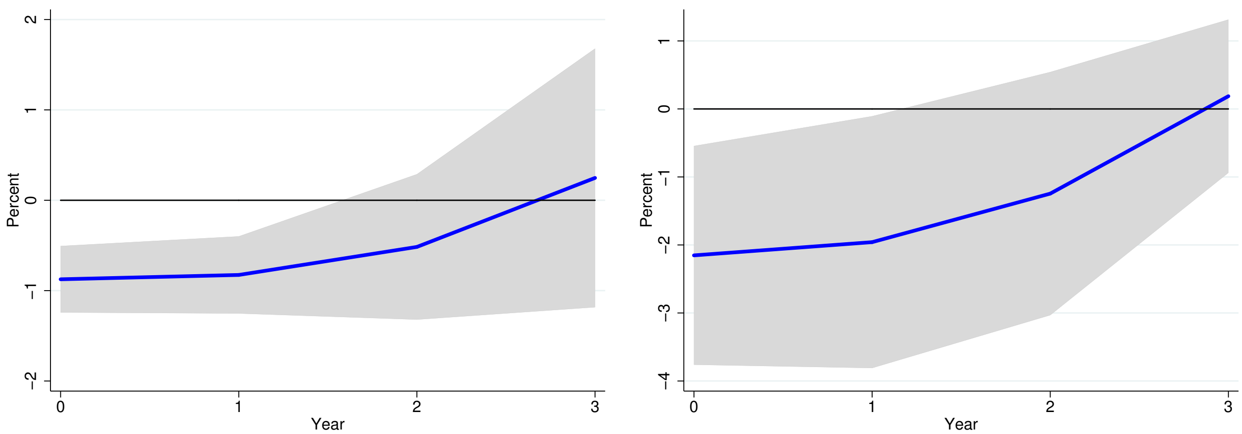

Figure 3: Response of Relative Labor of High markup Firm to Romer and Romer (2004) one standard error: Driscoll-Kraay standard errors are used. Shaded areas are 90% confidence intervals. The left chart shows the results for Chile for the period 1979-1996; the right chart shows the results for Colombia, for 1981-1991, both for the full sample. For markup, we use values obtained from the output elasticities of intermediate goods in Cobb-Douglas production functions obtained from the replication files in Raval (2019).

Figure 3 shows an impulse response of the relative log number of workers in high-markup firms in Chile and Colombia to the tightening of US monetary policy. In both countries, the relative decline is larger for high-markup firms over about two years, thus indicating a reallocation of labor from high-markup firms to low-markup firm. In appendix 2, we report the use of markup based on different specifications..

5.2 Macro Level Evidence

TBD

6 Conclusion

In this paper, we construct a model featuring misallocation due to heterogeneity in firm-level markup and the dollar as the dominant currency in trade. The tightening of US monetary policy leads to an appreciation in the dollar, which decreases the dollar-denominated marginal cost of non-US exporters, thus increasing the markup of large non-US firms and reducing allocative efficiency. In this case, allocative efficiency declines when large firms increase their markup. On the other hand, since the marginal cost that US firms refer to when setting their prices is in their home currency, the dollar, the effect of the exchange rate on the markup is limited. To test this mechanism, we use the identified monetary policy shocks to confirm that the micro-level reallocation and productivity responses are consistent with the theory.

The next fruitful direction is to develop a more realistic model that introduces intermediate goods transactions and explore its implications. It is also interesting to introduce realistic structures within the sector. For example, in the current model, all firms are exposed to the same trade exposure. However, in the trade literature, it is known that larger firms are more likely to export; introducing export selection would result in different exposures to external demand. Under different exposures to external demand shocks, endogenous external demand shocks associated with the appreciation in the dollar encourage reallocation from large non-US firms to smaller firms, exacerbating misallocation more directly. Finally, since our model is also applicable to HSA demand, it would be interesting to examine parametric functions belonging to HSA demand that satisfy Marshall's third law (e.g., Matsuyama and Ushchev (2022) has proposed a power elasticity of markup rate (PEM)). We are working on these extensions in ongoing work, and a revised version will be published soon.

References

Amiti, Mary, Oleg Itskhoki, and Jozef Konings, "International Shocks, Variable Markups, and Domestic Prices," *The Review of economic studies*, February 2019, *86* (6), 2356–2402.

_ , _ , and _ , "Dominant Currencies: How firms choose currency invoicing and why it matters," Technical Report 15339 October 2020.

Arkolakis, Costas, Arnaud Costinot, Dave Donaldson, and Andrés Rodríguez-Clare, "The Elusive Pro-Competitive Effects of Trade," *The Review of economic studies*, January 2018, *86* (1), 46–80.

Atkeson, Andrew and Ariel Burstein, "Pricing-to-Market, Trade Costs, and International Relative Prices," *The American economic review*, December 2008, *98* (5), 1998–2031.

Baqaee, David and Emmanuel Farhi, "Networks, Barriers, and Trade," Technical Report w26108, National Bureau of Economic Research July 2019.

__ , __ , **and Kunal Sangani**, "The Darwinian returns to scale," *SSRN Electronic Journal*, 2020.

__ , __ , **and** __ , "The Supply-Side Effects of Monetary Policy," Technical Report w28345, National Bureau of Economic Research January 2021.

Baqaee, David Rezza and Emmanuel Farhi, "Productivity and Misallocation in General Equilibrium," *The quarterly journal of economics*, February 2020, *135* (1), 105–163.

Bartelsman, Eric, John Haltiwanger, and Stefano Scarpetta, "Cross-Country Differences in Productivity: The Role of Allocation and Selection," *The American economic review*, February 2013, *103* (1), 305–334.

Basu, Susanto and John G Fernald, "Aggregate productivity and aggregate technology," *European economic review*, 2002, *46* (6), 963–991.

Boz, Emine, Camila Casas, Georgios Georgiadis, Gita Gopinath, Helena Le Mezo, Arnaud Mehl, and Tra Nguyen, "Patterns of invoicing currency in global trade: New evidence," *Journal of international economics*, May 2022, *136*, 103604.

Burstein, Ariel and Javier Cravino, "Measured Aggregate Gains from International Trade," *American Economic Journal: Macroeconomics*, April 2015, *7* (2), 181–218.

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber, "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins," 2011.

Edmond, Chris, Virgiliu Midrigan, and Daniel Yi Xu, "Competition, Markups, and the Gains from International Trade," *The American economic review*, 2015, *105* (10), 3183–3221.

Fujiwara, I and K Matsuyama, "Competition and the Phillips Curve," 2023.

Gaillard, Alexandre and François de Soyres, "Value-added and productivity linkages across countries," https://agaillardtse.github.io/projects/TCP.pdf 2021. Accessed: 2022-6-14.

Galí, Jordi, *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications - Second Edition*, Princeton University Press, June 2015.

Gopinath, Gita and Oleg Itskhoki, "Dominant Currency Paradigm: A Review," 2021.

__ **, Emine Boz, Camila Casas, Federico J Díez, Pierre-Olivier Gourinchas, and Mikkel Plagborg-Møller**, "Dominant Currency Paradigm," *The American economic review*, March 2020, *110* (3), 677–719.

Hall, Robert, "The Relation between Price and Marginal Cost in U.S. Industry," *The journal of political economy*, 1988, *96* (5), 921–947.

Hsieh, Chang-Tai and Peter J Klenow, "Misallocation and Manufacturing TFP in China and India," *The quarterly journal of economics*, 2009, *124* (4), 1403–1448.

Huo, Zhen, Andrei A Levchenko, and Nitya Pandalai-Nayar, "Utilization-adjusted TFP across countries: Measurement and implications for international comovement," *SSRN Electronic Journal*, 2020.

Itskhoki, Oleg, "The Story of the Real Exchange Rate," *Annual review of economics*, August 2021, *13* (1), 423–455.

— **and Dmitry Mukhin**, "Exchange Rate Disconnect in General Equilibrium," *The journal of political economy*, August 2021, *129* (8), 2183–2232.

Kimball, Miles S, "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of money, credit, and banking*, 1995, *27* (4), 1241–1277.

Klenow, Peter J and Jonathan L Willis, "Real rigidities and nominal price changes," *Economica*, July 2016, *83* (331), 443–472.

Matsuyama, Kiminori and Philip Ushchev, "Beyond CES: Three alternative classes of flexible homothetic demand systems," *SSRN Electronic Journal*, 2017.

— **and** — , "Selection and Sorting of Heterogeneous Firms through Competitive Pressure," 2022.

Mukhin, Dmitry, "An Equilibrium Model of the International Price System," *The American economic review*, February 2022, *112* (2), 650–688.

Raval, Devesh, "Testing the production approach to markup estimation," *SSRN Electronic Journal*, 2019.

Reinelt, Timo and Matthias Meier, "Monetary Policy, Markup Dispersion, and Aggregate TFP," June 2020.

Romer, Christina D and David H Romer, "A New Measure of Monetary Shocks: Derivation and Implications," *The American economic review*, September 2004, *94* (4), 1055–1084.

Appendix1: Non-Parametric Calibration of Sufficient Statistics

In identifying the Kimball demand function, we use the non-parametric methods of Baqaee et al. (2020)[9]. We will incorporate two objects in our data set, (1) the sales share density λ_θ and (2) the pass-through distribution ρ_θ. Using Belgian data, the distribution of pass-throughs by firm size uses values identified by Amiti et al. (2019). This is combined with the firm sales distribution to recover the markup.

They proved that the following differential equation holds between pass-through, markup, and firm size. We can recover the markup distribution by solving this differential equation under initial conditions. After recovering the markup distribution, we further recover the elasticity of substitution. We can recover the sufficient statistics needed using these two statistics, empirical pass-through and the firm distribution.

$$\frac{d\mu_\theta}{d\theta} = (\mathcal{M}_{\subseteq} - 1)\,\frac{1-\rho_\theta}{\rho_\theta}\frac{d\lambda_\theta}{d\theta}$$

The intuition of the idea is that since incomplete pass-through is due to incomplete pass-through resulting from changes in markup, we recover the distribution of markup by solving the above differential equation above that relates derivative of markup and pass-through. As an initial condition for markup, Baqaee et al. (2020) target a level of aggregate markup of 1.15, which we also follow. Table 3 reports the recovered value of sufficient statistics.

$E_{\lambda\sigma}[\rho_\theta]$,	$E_\lambda[\rho_\theta]$	$\frac{Cov_\lambda(\sigma_\theta,\rho_\theta)}{E_\lambda[\sigma_\theta]}$
0.78	0.65	0.2

Table 3: Recovered sufficient statistics for calibration

[9]Thanks to David Baqaee for kindly sharing their dataset.

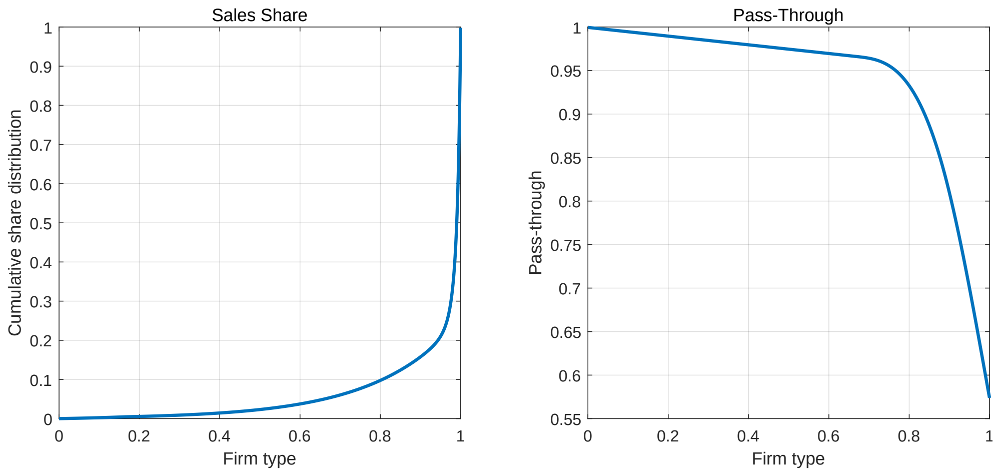

Figure 4: Recovered distribution of sales shares and pass-through.

Appendix2: Additional Empirical Result

Micro level results with translog

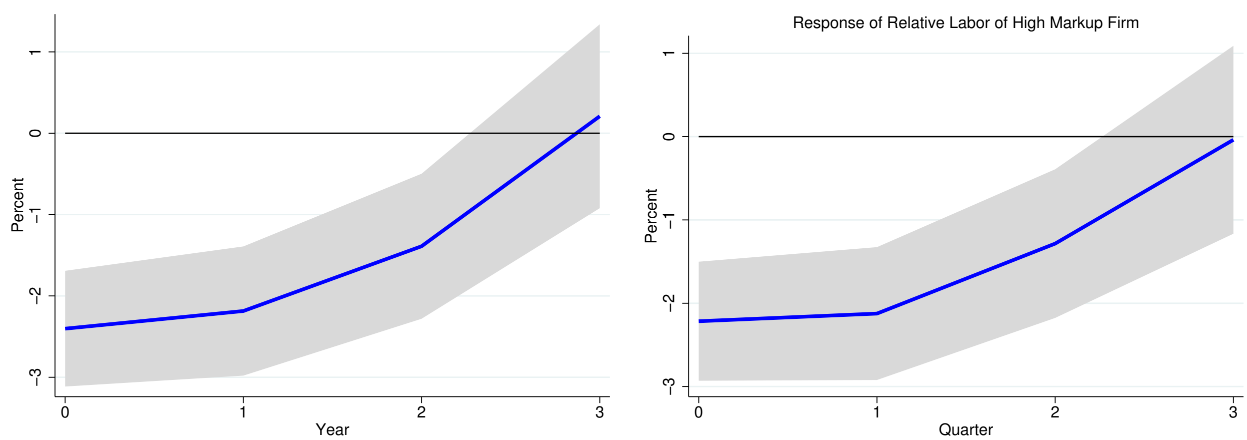

Figure 5: Response of relative labor of high-markup firm to Romar and Romer (2004) one standard error: Driscoll-Kraay standard errors are used. Shaded areas are 90% confidence intervals. The left figure shows the results for Chile for 1979-1996; the right chart shows the results for Colombia, for the period 1981-1991, both for the full sample. For markup, we use values obtained from the output elasticities of intermediate goods in Translog production functions obtained from the replication files in Raval (2019).

Appendix2: Proof

Proof of Lemma 1

Proof. Log-linear approximation of equation (4)

$$
\begin{aligned}
p_t &= p_t^{CPI} - \frac{\int \left[-\sigma_\theta \frac{p_\theta c_\theta}{PC} dlog \frac{c_\theta}{C} + \int \frac{p_\theta c_\theta}{PC} dlog \frac{c_\theta}{C}\right] d\theta}{\int \frac{p_\theta c_\theta}{PC} d\theta} \\
&= p_t^{CPI} - \frac{\int \left[1 - \frac{1}{\sigma_\theta}\right] \lambda_\theta dlog \frac{c_\theta}{C} d\theta}{\int \lambda_\theta d\theta} \\
&= p_t^{CPI} - (1-\gamma) E_\lambda \left[(1-\sigma_\theta)(p_{h\theta} - p)\right] - \gamma E_\lambda \left[(1-\sigma_\theta)\left(p_{f\theta} - p\right)\right] \\
&= p_t^{CPI} - p_t^{CPI} + E_\lambda [\sigma_\theta] \left[(1-\gamma) p_{ht}^{\sigma} + \gamma p_{ft}^{\sigma}\right] + (1 - E_\lambda [\sigma_\theta]) p_t \Leftrightarrow \\
p_t &= (1-\gamma) p_{ht}^{\sigma} + \gamma p_{ft}^{\sigma}
\end{aligned}
$$

$\square$

Proof of Lemma 2

Proof. Start from Theorem 1 of Baqaee and Farhi (2019) with no intermediate goods:

$$
\begin{aligned}
a_t &= Cov_\lambda \left[\left(\bar{\mathcal{M}}/\mathcal{M}_\theta\right) \mu_{h\theta t}\right] - Cov_\lambda \left[\left(\bar{\mathcal{M}}/\mathcal{M}_\theta\right) \lambda_{h\theta t}\right] \\
&= Cov_\lambda \left[\left(\bar{\mathcal{M}}/\mathcal{M}_\theta\right) \left[(1-\gamma) \mu_{h\theta t}^{home} + \gamma \mu_{\theta t}^{*}\right]\right] - Cov_\lambda \left[\left(\bar{\mathcal{M}}/\mathcal{M}_\theta\right) \left[(1-\gamma) \lambda_{h\theta t}^{home} + \gamma \lambda_{\theta t}^{*}\right]\right] \\
&= Cov_\lambda \left[-\left(\bar{\mathcal{M}}/\mathcal{M}_\theta\right) \left[(1-\gamma)(\lambda_{h\theta t} - \mu_{h\theta t}) + \gamma (\lambda_{h\theta t}^{*} - \mu_{\theta t}^{*})\right]\right] \\
&= Cov_\lambda \left[-\left(\bar{\mathcal{M}}/\mathcal{M}_\theta\right), (1-\gamma) l_{h\theta t} + \gamma l_{h\theta t}^{*}\right] \\
&= Cov_\lambda \left[-\left(\bar{\mathcal{M}}/\mathcal{M}_\theta\right), totalcost_{h\theta t}\right]
\end{aligned}
$$

$\square$

Proof of Proposition 1

Proof. Substitute the demand function into Lemma 2 and rearrange

$$
\begin{aligned}
a_t =& Cov_\lambda \left[-\left(\bar{\mathcal{M}} / \bar{\mathcal{M}}_\theta \right), \{ (1-\gamma)\, l_{h\theta t} + \gamma l^*_{h\theta t} \} \right] \\
=& Cov_\lambda \left[-\left(\bar{\mathcal{M}} / \bar{\mathcal{M}}_\theta \right), \{ (1-\gamma)\, c_{h\theta t} + \gamma c^*_{h\theta t} \} \right] \\
=& \bar{\mathcal{M}} Cov_\lambda \left[(1/\mathcal{M}_\theta), \sigma_\theta \{ (1-\gamma) (p_{h\theta t} - p_t) + \gamma (p^*_{h\theta t} - p^*_t) \} \right] \\
=& \bar{\mathcal{M}} E_\lambda \left[(\sigma_\theta - 1) \{ (1-\gamma) (p_{h\theta t} - p_t) + \gamma (p^*_{h\theta t} - p^*_t) \} \right] \\
& - E_\lambda \left[\sigma_\theta \{ [(1-\gamma) (p_{h\theta t} - p_t) + \gamma (p^*_{h\theta t} - p^*_t)] \} \right] \\
=& \left(\bar{\mathcal{M}} - 1 \right) E_\lambda [\sigma_\theta] \left[(1-\gamma) (p^\sigma_{ht} - p_t) + \gamma (p^{\sigma *}_{ht} - p^*_t) \right] \\
& - \bar{\mathcal{M}} \left[(1-\gamma) (p_{ht} - p_t) + \gamma (p^*_{ht} - p^*_t) \right]
\end{aligned}
$$

Thus,

$$
a_t = \left(\bar{\mathcal{M}} - 1 \right) E_\lambda [\sigma_\theta] \left[(1-\gamma) (p^\sigma_{ht} - p_t) + \gamma (p^{\sigma *}_{ht} - p^*_t) \right] - \bar{\mathcal{M}} \left[(1-\gamma) (p_{ht} - p_t) + \gamma (p^*_{ht} - p^*_t) \right]
$$

where

$$
p_t = (1-\gamma)\, p^\sigma_{ht} + \gamma p^\sigma_{ft}
$$

$$
p^*_t = (1-\gamma)\, p^{\sigma *}_{ht} + \gamma p^{\sigma *}_{ft}
$$

The dynamics of the price index is obtained by aggregating the price dynamics of individual firms with different weights.

$$
p_{ht} = \frac{\kappa}{\psi} p_{ht-1} + \frac{\kappa\beta}{\psi} p_{ht+1} + \kappa \left(E_\lambda [\rho_\theta]\, w_t + (1 - E_\lambda [\rho_\theta])\, p_t \right)
$$

$$
p^*_{ht} = \frac{\kappa}{\psi} p^*_{ht-1} + \frac{\kappa\beta}{\psi} p^*_{ht+1} + \kappa \left(E_\lambda [\rho_\theta] [w_t - e_t] + (1 - E_\lambda [\rho_\theta])\, p^*_t \right)
$$

$$
p_{ft} - e_t = \frac{\kappa}{\psi} \left(p_{ft-1} - e_{t-1} \right) + \frac{\kappa\beta}{\psi} \left(p_{ft+1} - e_{t+1} \right) + \kappa \left(E_\lambda [\rho_\theta]\, w^*_t + (1 - E_\lambda [\rho_\theta]) (p_t - e_t) \right)
$$

$$
p^*_{ft} = \frac{\kappa}{\psi} p^*_{ft-1} + \frac{\kappa\beta}{\psi} p^*_{ft+1} + \kappa \left(E_\lambda [\rho_\theta]\, w^*_t + (1 - E_\lambda [\rho_\theta])\, p^*_t \right)
$$

Sigma weighed price:

$$
p^\sigma_{ht} = \frac{\kappa}{\psi} p^\sigma_{ht-1} + \frac{\kappa\beta}{\psi} p^\sigma_{ht+1} + \kappa \left(E_{\lambda\sigma} [\rho_\theta]\, w_t + (1 - E_{\lambda\sigma} [\rho_\theta])\, p_t \right)
$$

$$p_{ht}^{\sigma *}=\frac{\kappa}{\psi}p_{ht-1}^{\sigma *}+\frac{\kappa\beta}{\psi}p_{ht+1}^{\sigma *}+\kappa\left(E_{\lambda\sigma}\left[\rho_{\theta}\right]\left[w_t-e_t\right]+\left(1-E_{\lambda\sigma}\left[\rho_{\theta}\right]\right)p_t^{*}\right)$$

$$p_{ft}^{\sigma}-e_t=\frac{\kappa}{\psi}\left(p_{ft}^{\sigma}-e_{t-1}\right)+\frac{\kappa\beta}{\psi}\left(p_{ft}^{\sigma}-e_{t+1}\right)+\kappa\left(E_{\lambda\sigma}\left[\rho_{\theta}\right]w_t^{*}+\left(1-E_{\lambda\sigma}\left[\rho_{\theta}\right]\right)\left(p_t-e_t\right)\right)$$

$$p_{ft}^{\sigma *}=\frac{\kappa}{\psi}p_{ft-1}^{\sigma *}+\frac{\kappa\beta}{\psi}p_{ft+1}^{\sigma *}+\kappa\left(E_{\lambda\sigma}\left[\rho_{\theta}\right]w_t^{*}+\left(1-E_{\lambda\sigma}\left[\rho_{\theta}\right]\right)p_t^{*}\right)$$

Combining these price definitions with the productivity equation, we obtain the following expression

$$\begin{aligned}
a_t &= \frac{\kappa}{\psi}a_{t-1}+\frac{\kappa\beta}{\psi}a_{t+1}\\
&+\kappa\left(\underbrace{2\left(1-\gamma\right)\gamma\left(\bar{\mathcal{M}}-1\right)E_{\lambda}\left[\sigma_{\theta}\right]E_{\lambda\sigma}\left[\rho_{\theta}\right]-\bar{\mathcal{M}}E_{\lambda}\left[\rho_{\theta}\right]+\bar{\mathcal{M}}E_{\lambda\sigma}\left[\rho_{\theta}\right]\left[\left(1-\gamma\right)^{2}+\gamma^{2}\right]}_{(*)}\right)w_t\\
&+\kappa\left(-\left(E_{\lambda}\left[\sigma_{\theta}\right]\left(\bar{\mathcal{M}}-1\right)-\bar{\mathcal{M}}\right)E_{\lambda\sigma}\left[\rho_{\theta}\right]2\gamma\left(1-\gamma\right)\right)w_t^{*}\\
&+\kappa\left(\underbrace{E_{\lambda}\left[\sigma_{\theta}\right]\left(1-E_{\lambda\sigma}\left[\rho_{\theta}\right]\right)\left(\left(\bar{\mathcal{M}}-1\right)-\left(\bar{\mathcal{M}}-1\right)+\frac{\bar{\mathcal{M}}}{E_{\lambda}\left[\sigma_{\sigma}\right]}\right)-\bar{\mathcal{M}}\left(1-E_{\lambda}\left[\rho_{\theta}\right]\right)}_{(**)}\right)\left[\left(1-\gamma\right)p_t+\gamma p_t^{*}\right]\\
&+\kappa\left(\underbrace{\begin{matrix}-\left(\bar{\mathcal{M}}-1\right)E_{\lambda}\left[\sigma_{\theta}\right]E_{\lambda\sigma}\left[\rho_{\theta}\right]+\bar{\mathcal{M}}E_{\lambda}\left[\rho_{\theta}\right]+\\ \left(\bar{\mathcal{M}}-E_{\lambda}\left[\sigma_{\theta}\right]\left(\bar{\mathcal{M}}-1\right)\right)\left[\left(\frac{1+\beta}{\psi}+E_{\lambda\sigma}\left[\rho_{\theta}\right]\right)\left(1-\gamma\right)-E_{\lambda\sigma}\left[\rho_{\theta}\right]\gamma\right]\end{matrix}}_{(***)}\right)\gamma e_t\\
&-\kappa\frac{\left(1-\gamma\right)\gamma}{\psi}\left(\bar{\mathcal{M}}-E_{\lambda}\left[\sigma_{\theta}\right]\left(\bar{\mathcal{M}}-1\right)\right)\left[e_{t-1}+\beta e_{t+1}\right]
\end{aligned}$$

where $\kappa=\frac{\psi}{1+\beta+\psi}$.

We know that

$$\begin{aligned}
\left(E_{\lambda}\left[\sigma_{\theta}\right]\left(\bar{\mathcal{M}}-1\right)-\bar{\mathcal{M}}\right) &= E_{\lambda}\left[\sigma_{\theta}\right]\left(\frac{E_{\lambda}\left[\frac{1}{\sigma_{\theta}}\right]}{1-E_{\lambda}\left[\frac{1}{\sigma_{\theta}}\right]}\right)-\left(\frac{1}{1-E_{\lambda}\left[\frac{1}{\sigma_{\theta}}\right]}\right)\\
&= \bar{M}\left(E_{\lambda}\left[\sigma_{\theta}\right]E_{\lambda}\left[\frac{1}{\sigma_{\theta}}\right]-1\right)\\
&= \bar{\mathcal{M}}Cov_{\lambda}\left[\sigma_{\theta},\frac{1}{\mathcal{M}_{\theta}}\right]>0
\end{aligned}$$

Then,

$$\begin{aligned}(*) =& 2\left(1-\gamma\right)\gamma\left(\bar{\mathcal{M}}-1\right)E_{\lambda}\left[\sigma_{\theta}\right]E_{\lambda\sigma}\left[\rho_{\theta}\right]-\bar{\mathcal{M}}E_{\lambda}\left[\rho_{\theta}\right]+\bar{\mathcal{M}}E_{\lambda\sigma}\left[\rho_{\theta}\right]\left[\left(1-\gamma\right)^{2}+\gamma^{2}\right]\\
=&\bar{\mathcal{M}}\left(E_{\lambda\sigma}\left[\rho_{\theta}\right]-E_{\lambda}\left[\rho_{\theta}\right]\right)-\left(\bar{\mathcal{M}}-E_{\lambda}\left[\sigma_{\theta}\right]\left(\bar{\mathcal{M}}-1\right)\right)E_{\lambda\sigma}\left[\rho_{\theta}\right]2\gamma\left[1-\gamma\right]\\
=&\bar{\mathcal{M}}Cov_{\lambda}\left(\frac{\sigma_{\theta}}{E_{\lambda}\left[\sigma_{\sigma}\right]},\rho_{\theta}\right)+Cov_{\lambda}\left(\frac{\bar{\mathcal{M}}}{\mathcal{M}_{\theta}},\sigma_{\theta}\right)E_{\lambda\sigma}\left[\rho_{\theta}\right]2\gamma\left[1-\gamma\right]\end{aligned}$$

$$\begin{aligned}(**) &= \left(1-E_{\lambda\sigma}\left[\rho_{\theta}\right]\right)\bar{\mathcal{M}}-\bar{\mathcal{M}}\left(1-E_{\lambda}\left[\rho_{\theta}\right]\right)\\
&=-\bar{\mathcal{M}}Cov_{\lambda}\left(\frac{\sigma_{\theta}}{E_{\lambda}\left[\sigma_{\sigma}\right]},\rho_{\theta}\right)<0\end{aligned}$$

$$\begin{aligned}(***) &= -\bar{\mathcal{M}}\frac{Cov_{\lambda}\left(\sigma_{\theta}\rho_{\theta}\right)}{E_{\lambda}\left[\sigma_{\sigma}\right]}\gamma+\left(\bar{\mathcal{M}}-E_{\lambda}\left[\sigma_{\theta}\right]\left(\bar{\mathcal{M}}-1\right)\right)\left[2E_{\lambda\sigma}\left[\rho_{\theta}\right]\left(1-\gamma\right)+\frac{1+\beta}{\psi}\left(1-\gamma\right)\right]\gamma\\
&=-\bar{\mathcal{M}}\frac{Cov_{\lambda}\left(\sigma_{\theta}\rho_{\theta}\right)}{E_{\lambda}\left[\sigma_{\sigma}\right]}\gamma-Cov_{\lambda}\left(\frac{\bar{\mathcal{M}}}{\mathcal{M}_{\theta}},\sigma_{\theta}\right)\left[2E_{\lambda\sigma}\left[\rho_{\theta}\right]\left(1-\gamma\right)+\frac{1+\beta}{\psi}\left(1-\gamma\right)\right]\gamma\\
&=-\chi\gamma+\xi-\omega\end{aligned}$$

Define

$$\chi=\bar{\mathcal{M}}\frac{Cov_{\lambda}\left(\sigma_{\theta},\rho_{\theta}\right)}{E_{\lambda}\left[\sigma_{\theta}\right]}$$

,

$$\xi=2\gamma\left(1-\gamma\right)E_{\lambda\sigma}\left[\rho_{\theta}\right]\bar{\mathcal{M}}Cov_{\lambda}\left[\sigma_{\theta},\frac{1}{\mathcal{M}_{\theta}}\right]>0$$

$$\omega=\frac{1}{\psi}\left(1-\gamma\right)\gamma\bar{\mathcal{M}}Cov_{\lambda}\left[\sigma_{\theta},\frac{1}{\mathcal{M}_{\theta}}\right]>0$$

Rearrange to achieve desired results.

$$\begin{aligned}a_{t}=&\frac{\kappa}{\psi}a_{t-1}+\frac{\kappa}{\psi}\beta a_{t+1}\\
&+\kappa\left\{\begin{array}{c}\left(\chi-\xi\right)w_{t}+\xi w_{t}^{*}-\chi\left[\left(1-\gamma\right)p_{t}+\gamma p_{t}^{*}\right]-\left[\chi\gamma-\xi\right]e_{t}\\
-\omega\left[\Delta e_{t}-\beta\Delta e_{t+1}\right]\end{array}\right\}\end{aligned}$$

The same calculation is performed for US Productivity.

$$
\begin{aligned}
a_t^* &= \frac{\kappa}{\psi}a_{t-1}^* + \frac{\kappa\beta}{\psi}a_{t+1}^* \\
&+ \kappa\left\{(\chi-\xi)\,{w^*}_t + \xi w_t - \chi\left[(1-\gamma)\,p_t^* + \gamma p_t\right]\right\} \\
&+ \kappa\left(\underbrace{\bar{\mathcal{M}}\frac{Cov_\lambda\left(\sigma_\theta\rho_\theta\right)}{E_\lambda\left[\sigma_\theta\right]} + \left(\bar{\mathcal{M}} - E_\lambda\left[\sigma_\theta\right]\left(\bar{\mathcal{M}}-1\right)\right)\left[-\left(\frac{1+\beta}{\psi} + E_{\lambda\sigma}\left[\rho_\theta\right]\right) - E_{\lambda\sigma}\left[\rho_\theta\right]\right]}_{(*)}(1-\gamma)\right)\gamma e_t \\
&- \frac{\kappa}{\psi}\left(\left(\bar{\mathcal{M}}-1\right)E_\lambda\left[\sigma_\theta\right] - \bar{\mathcal{M}} + \gamma\left(\bar{\mathcal{M}} - E_\lambda\left[\sigma_\theta\right]\left(\bar{\mathcal{M}}-1\right)\right)\right)\gamma e_{t-1} \\
&- \frac{\kappa\beta}{\psi}\left(\left(\bar{\mathcal{M}}-1\right)E_\lambda\left[\sigma_\theta\right] - \bar{\mathcal{M}} + \gamma\left(\bar{\mathcal{M}} - E_\lambda\left[\sigma_\theta\right]\left(\bar{\mathcal{M}}-1\right)\right)\right)\gamma e_{t+1}
\end{aligned}
$$

where

$$
(*) = [\chi\gamma - \xi] + \omega\frac{(1-\gamma)\,\gamma}{\psi}
$$

Therefore, we have

$$
\begin{aligned}
a_t^* &= \frac{\kappa}{\psi}a_{t-1}^* + \frac{\kappa\beta}{\psi}a_{t+1}^* \\
&+ \kappa\left\{(\chi-\xi)\,{w^*}_t + \xi w_t - \chi\left[(1-\gamma)\,p_t^* + \gamma p_t\right] + [\chi\gamma-\xi]\,e_t + \omega\left[\Delta e_t - \beta\Delta e_{t+1}\right]\right\}
\end{aligned}
$$

□

Proof of Lemma 4

Proof. Let (10)$= F$ and apply implicit function theorem to obtain $\frac{\partial \log P}{\partial \log P_{h\theta}}$:

$$
\begin{aligned}
\frac{\partial \log P}{\partial \log P_{h\theta}} &= -\frac{\frac{\partial \log F}{\partial P_{h\theta}}P_{h\theta}}{\frac{\partial \log F}{\partial P}P} \\
&= \frac{(1-\gamma)\,\frac{1}{P}s_\theta'\left(\frac{p_{h\theta}}{P}\right)}{\int\left[(1-\gamma)\,\frac{p_{h\theta}}{P^2}s_\theta'\left(\frac{p_{h\theta}}{P}\right) + \frac{p_{f\theta}}{P^2}\gamma s_\theta'\left(\frac{p_{f\theta}}{\gamma P}\right)\right]d\theta}\frac{P}{P_{h\theta}} \\
&= \frac{(1-\gamma)\,\frac{p_{h\theta}}{P}s_\theta'\left(\frac{p_{h\theta}}{P}\right)}{\int\left[(1-\gamma)\,\frac{p_{h\theta}}{P}s_\theta'\left(\frac{p_{h\theta}}{P}\right) + \frac{p_{f\theta}}{P}\gamma s_\theta'\left(\frac{p_{f\theta}}{\gamma P}\right)\right]d\theta}
\end{aligned}
$$

$$\frac{\partial \log P}{\partial \log P_{f\theta}} = \frac{(1-\gamma)\frac{p_{f\theta}}{P}s'_{\theta}\left(\frac{p_{f\theta}}{P}\right)}{\int\left[(1-\gamma)\frac{p_{h\theta}}{P}s'_{\theta}\left(\frac{p_{h\theta}}{P}\right)+\frac{p_{f\theta}}{P}\gamma s'_{\theta}\left(\frac{p_{f\theta}}{\gamma P}\right)\right]d\theta}$$

Therefore,

$$\begin{aligned}
d\log P &= \int\left(\frac{\partial \log P}{\partial \log P_{h\theta}}d\log P_{h\theta}+\frac{\partial \log P}{\partial \log P_{f\theta}}d\log P_{f\theta}\right)d\theta \\
&= \int \frac{(1-\gamma)\frac{p_{f\theta}}{P}s'_{\theta}\left(\frac{p_{f\theta}}{P}\right)d\log P_{h\theta t}+\frac{p_{f\theta}}{P}\gamma s'_{\theta}\left(\frac{p_{f\theta}}{\gamma P}\right)d\log P_{f\theta}}{\int\left[(1-\gamma)\frac{p_{h\theta}}{P}s'_{\theta}\left(\frac{p_{h\theta}}{P}\right)+\frac{p_{f\theta}}{P}\gamma s'_{\theta}\left(\frac{p_{f\theta}}{\gamma P}\right)\right]d\theta}d\theta \Leftrightarrow \\
p_t &= \int \frac{s_{\theta}(1-\sigma_{\theta})\left[(1-\gamma)p_{h\theta t}+\gamma p_{f\theta t}\right]}{\int\left[s_{\theta}(1-\sigma_{\theta})\right]d\theta} \\
&= \frac{E_{\lambda}[\sigma_{\theta}]\left((1-\gamma)p^{\sigma}_{ht}+\gamma p^{\sigma}_{ft}\right)-p^{CPI}_t}{(E_{\lambda}[\sigma_{\theta}]-1)}
\end{aligned}$$

We used demand elasticity $\frac{p_{h\theta t}}{P_t}s'_{\theta}=s_{\theta}(1-\sigma_{\theta})$. □

Appendix 3: Log Liniearlized System

Allocative Efficiency

$$\begin{aligned}
a_t &= \frac{\kappa}{\psi}a_{t-1}+\frac{\kappa}{\psi}\beta a_{t+1} \\
&+\kappa\left\{\begin{array}{c}(\chi+\xi)w_t+\xi w^*_t-\chi\left[(1-\gamma)p_t+\gamma p^*_t\right]-\left[\chi\gamma-\xi\right]e_t \\ +\omega\left[\Delta e_t-\beta\Delta e_{t+1}\right]\end{array}\right\}
\end{aligned}$$

$$\begin{aligned}
a^*_t &= \frac{\kappa}{\psi}a^*_{t-1}+\frac{\kappa\beta}{\psi}a^*_{t+1} \\
&+\kappa\left\{\begin{array}{c}(\chi+\xi){w^*}_t+\xi w_t-\chi\left[(1-\gamma)p^*_t+\gamma p_t\right]+\left[\chi\gamma-\xi\right]e_t \\ -\omega\left[\Delta e_t-\beta\Delta e_{t+1}\right]\end{array}\right\}
\end{aligned}$$

where $p_t=(1-\gamma)E_{\lambda\sigma}\left[p_{\theta H,t}\right]+\gamma E_{\lambda\sigma}\left[p_{\theta F,t}\right]$ and $p^*_t=(1-\gamma)E_{\lambda\sigma}\left[p^*_{\theta F,t}\right]+\gamma E_{\lambda\sigma}\left[p^*_{\theta H,t}\right]$.

Symmetric Equations (Home and Foreign counterparts)

Households

$$i_t=\eta\left(\mathbb{E}_t c_{t+1}-c_t\right)+\pi^{CPI}_{t+1} \tag{11}$$

$$\eta c_t + \zeta l_t = w_t - p_t^{CPI} \tag{12}$$

where $p_t^{CPI} = (1-\gamma)\, p_{Ht} + \gamma p_{Ft}$ and $p_t^{CPI*} = (1-\gamma)\, p_{Ft}^{*} + \gamma p_{Ht}^{*}$

Monetary policies:

$$i_t = \rho_m i_{t-1} + (1-\rho_m)\, \phi_\pi \pi_t^{CPI} \tag{13}$$

Market Clearing + Budget constraint:

$$y_t = -\gamma \left[e_t + \left(p_h^* - p_f\right)\right] + (\beta b_{t+1} - b_t) + c_t \tag{14}$$

NKPC:

$$\pi_{Ht} = \psi\left(-p_{H,t} + E_\lambda\left[\rho_\theta\right] w_t + \left(1 - E_\lambda\left[\rho_\theta\right]\right) p_t\right) + \beta \mathbb{E}_t \pi_{Ht+1} \tag{15}$$

$$\pi_{Ht}^* = \psi\left(-p_{H,t}^* + E_\lambda\left[\rho_\theta\right](w_t - e_t) + \left(1 - E_\lambda\left[\rho_\theta\right]\right) p_t^*\right) + \beta \mathbb{E}_t \pi_{Ht+1}^* \tag{16}$$

$$\pi_{Ft} - \Delta e_t = \psi\left(-\left[p_{F,t} - e_t\right] + E_\lambda\left[\rho_\theta\right] w_t^* + \left(1 - E_\lambda\left[\rho_\theta\right]\right) p_t\right) + \beta \mathbb{E}_t\left(\pi_{Ft+1} - \Delta e_{t+1}\right) \tag{17}$$

$$\pi_{Ft}^* = \psi\left(-p_{F,t}^* + E_\lambda\left[\rho_\theta\right] w_t^* + \left(1 - E_\lambda\left[\rho_\theta\right]\right) p_t^*\right) + \beta \mathbb{E}_t \pi_{Ft+1}^* \tag{18}$$

where $\pi_{Xt} = p_{Xt} - p_{Xt-1}$ and $p_{Xt} = E_\lambda\left[p_{X\theta t}\right]$

Price Indices for Competition:

$$\pi_t - \gamma \Delta e_t = \psi\left(E_{\lambda\sigma}\left[\rho_\theta\right]\left[(1-\gamma)\, w_t + \gamma\left(w_t^* + e_t\right) - p_t\right]\right) + \beta\left[\mathbb{E}_t \pi_{t+1} - \gamma \Delta e_{t+1}\right] \tag{19}$$

$$\pi_t^* = \psi\left(E_{\lambda\sigma}\left[\rho_\theta\right]\left[(1-\gamma)\, w_t^* + \gamma\left(w_t + e_t\right) - p_t^*\right]\right) + \beta \mathbb{E}_t \pi_{t+1} \tag{20}$$

International:

$$i_t - i_t^* - \left[e_{t+1} - e_t\right] = 0 \tag{21}$$